Exam January 2021

1. 50 grams of Water is contained in a 1 m^3 rigid cylinder at 30◦C. if the condition is a saturated mixture,
    a) What is the pressure and the specific volume of the vapor? [1 mark]
    b) What will be the mass of the vapor alone if we neglected the volume of the liquid water? And what is the quality of the mixture? [1 mark]

Solution Q1

a) At 30◦C since liquid and vapor are coexisting at equilibrium then the pressure is the saturation pressure at 30◦C , so from the steam tables Ps@ 30◦C = 0.04242 bar and the specific volume of the vapor is = 32.93 m^3/kg
b) The mass of the vapor will be = 1/32.93 = 0.030367 kg, the quality will be x=0.030367/0.05 = 0.607 = 60.7%

2. For the water in Q1, what will be the temperature and pressure inside the cylinder if the cylinder is heated until all the water is 100% saturated steam? [2 marks]

When all the water is converted to steam, specific volume is = 1/0.05= 20 m3/kg,

interpolating in the steam table ,

32	0.04754	29.57
34	0.05318	26.60
36	0.05940	23.97
38	0.06624	21.63
40	0.07375	19.55
42	0.08198	17.69
44	0.09100	16.03
46	0.1009	14.56

we get,

$T = 40 + \frac{20-19.55}{21.63-19.55}(38-40) = 39.57°C$, and the pressure is

$P = 0.07375 + \frac{20-19.55}{21.63-19.55}(0.06624 - 0.07375) = 0.0721 bar$

3. Atmospheric air at 1.015 bar and of 80% relative humidity and a temperature of 30◦C is flowing into an air conditioning duct at a volume flow rate of 1.5 m^3/s. It is required to deliver this air into a room at a condition of 24°C and 50% relative humidity. The mass flow rate of the dry air is 1.75 kg/s.

Calculate the specific humidity at inlet. Given the mass flow of vapour in the vapour in air at outlet is 0.0093 kg/kg, calculate the mass flow rate of condensate.

Specific humidity $\omega = 0.622 \frac{p_s}{p_a - p_s}, \frac{p_s}{p_g} = 0.8 \rightarrow p_s = 0.8 p_g, at\ 30°C\ p_g = 0.04242\ bar$

$p_s = 0.8 * 0.04242 = 0.03393\ bar$ [1]

$\omega = 0.622 \frac{p_s}{p_a - p_s} = 0.622 \frac{0.03393}{0.98107} = 0.0215\ kg\ moisture\ /kg\ dry\ air$

$\dot{m}_{v-inlet} = \omega\, \dot{m}_{air} = 0.0215 * 1.75 = 0.0376\ kg/s$ [1]

$\dot{m}_{v-exit} = \omega\, \dot{m}_{air} = 0.00927 * 1.75 = 0.01622\ kg/s$ [1]

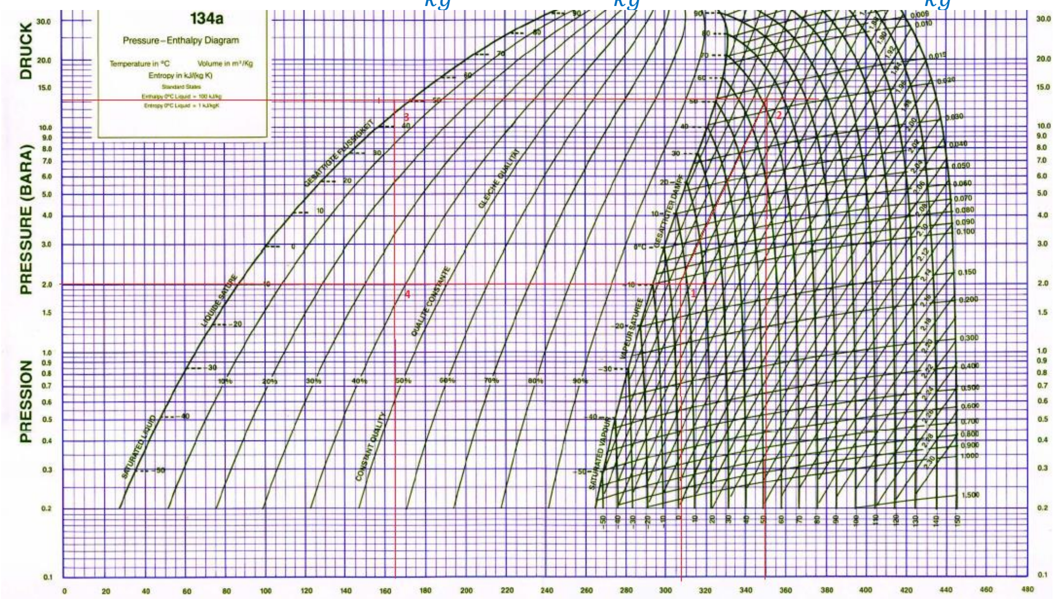
$\dot{m}_{condensate} = \dot{m}_{v-inlet} - \dot{m}_{v-exit} = 0.0376 - 0.01622 = 0.0214\ kg/s$ [1]

4. A heat pump is to be used for cooling a certain space using R134A as a refrigerant, assuming the compressor is isentropic, and that the evaporator exit has 5°C superheat and the condenser has 7°C subcooling.
    a) State the required pressure of the refrigerant in each of the condenser and the evaporator if a temperature of each is 50∘C and -10∘C respectively? [2]
    b) use the chart to find the enthalpy change across the evaporator and hence the mass flow rate of the refrigerant required in the system if heat is to be removed from the room at a rate of 3 kW? [2]

Solution Q4

a) Using the P-h diagram of R134A the saturation pressure at the condenser is, $p_{cond} = p_{sat@50°C} = 13.18\ bar$ $\ and\ p_{evapo.} = p_{sat@-10°C} = 2.01\ bar$ (to the correct single unit required, so 13 and 2 bar is sufficient). [1]

b) From the p-h diagram $h_1 = 308\frac{kJ}{kg},\ h_2 = 350\frac{kJ}{kg}\ and\ h_3 = h_4 = 165\frac{kJ}{kg}$



[1]

$$\dot{Q}_{cooling} = 3kW = \dot{m}(h_1 - h_4)$$

$$\rightarrow\ \dot{m} = \frac{3}{308-165} = 0.021\ kg/s$$ [1]

5. vapour power

Steam enters a turbine at 40 bar and 500°C and leaves at 10 bar. The mass flow rate of the steam is 20 kg·s^{-1} and the isentropic efficiency of the steam turbine is 94%. Calculate the power out of the turbine using the data tables.

Enthalpy in is p.7 of tables, h_{in} = 3445 kJ/kg [0.5]

entropy in is p.7 of tables, s_{in} = 7.089 kJ/kgK [0.5]

At 10 bar this entropy appears between 250 and 300°C, therefore interpolate to get $h_{out,isentropic}$

$\frac{h-2944}{3052-2944} = \frac{7.082-6.926}{7.124-6.926} \to h = 3033\ kJ/kg$ [1]

Use isentropic efficiency formula to get the actual h_{out}

$\eta_i = \frac{\Delta h_{actual}}{\Delta h_{isentropic}} = \frac{3445-h_{out,actual}}{3445-3033} = 0.94 \to h_{out,actual} = 3058\ kJ/kg$ [1]

Therefore power is by SFEE:

$\dot{W} = 20 \times (3445 - 3058) = 7760\ kW$ [1]

i.e. 7.76 MW

6. compressors

A reciprocating air compressor has a piston diameter 0.1 m and stroke 0.1 m, and the clearance volume is 10% of the stroke volume. The polytropic coefficient of the compression and expansion strokes is 1.2, the outflow pressure is 10 bar. Calculate the mass flow rate for the speed of 180 strokes per minute, and comment on the outcome of the performance you see. The ingoing air is at 1 bar and 288 K and has a density of 1.21 kg/m^3.

Get mass rate by formula: $\dot{m} = N\rho\eta_{vol}V_{swept}$

Get volumetric efficiency by formula: $\eta_{vol} = 1 - \frac{V_C}{V_S}\left\{\left[\frac{p_2}{p_1}\right]^{\frac{1}{n}} - 1\right\}$

The stroke volume is: $\frac{\pi}{4}0.1^3 = 0.00079$ m^3 [0.5]

The volumetric efficiency is: $\eta_{vol} = 1 - 0.1\left\{\left[\frac{10}{1}\right]^{\frac{1}{1.2}} - 1\right\} = 0.42$ [1.5]

mass flow rate is: $\dot{m} = 3 \times 1.21 \times 0.42 \times 0.00079 = 0.0012\ kg/s$ [1]

The volumetric efficiency is very low, which could be improved by using two stage compression, but it would make the machine more complicated. [1]

7. For the flow to be incompressible, we need that:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0.$$

We first find the partial derivatives:

$$\frac{\partial u}{\partial x} = 2xy + bax^{b-1}y^c, \qquad \frac{\partial v}{\partial y} = 1 - 2xy,$$

[2]

For the continuity equation to be satisfied, we need:

$$2xy + bax^{b-1}y^c + 1 - 2xy = 0 \implies bax^{b-1}y^c = -1,$$

Therefore we need $ab = -1, b - 1 = 0, c = 0$, which requires that:

$$a = -1,$$

$$b = 1,$$

$$c = 0.$$

[2]

8. With the simplifications (i)-(v), the 2D Navier-Stokes equations for an incompressible flow can be reduced to the solution of the x-momentum equation written as:

$$\frac{d^2u}{dy^2} = \frac{1}{\mu}\frac{dp}{dx},$$

To be solved between (bottom wall) $y = 0$ and (top wall) $y = H$ with boundary conditions:

$$u(y = 0) = 0,$$

$$u(y = H) = U_w.$$

Integrating the x-momentum equation twice:

$$u(y) = \frac{1}{2\mu}\frac{dp}{dx}y^2 + ay + b$$

[1]

Imposition of the boundary conditions yields:

$$b = 0,$$

$$a = \frac{U_w}{H} - \frac{1}{\mu}\frac{dp}{dx}\frac{H}{2},$$

And thus the theoretical velocity profile that responds to question (a) is:

$$u(y) = \frac{1}{2\mu}\frac{dp}{dx}(y^2 - Hy) + \frac{U_w}{H}y.$$

[1]

Extension:

U_W =0.5 m/s, H = 0.01 m, μ=0.001 kg/ms, dp/dx = 5 Pa/m, velocity at 0.007 is:

$$u(y) = \frac{1}{2 \times 0.001}(-5)(0.007^2 - 0.01 \times 0.007) + \frac{0.5}{0.01}0.007 = 0.053 + 0.350$$
$$= 0.403\ m/s$$

[2]

9. The flow becomes fully developed where the two boundary layers on the top and bottom walls join, therefore when $\delta = H/2$. Using Blasius equation for the thickness of a laminar boundary layer, we have:

$$\delta = \frac{5x}{\sqrt{\frac{\rho U x}{\mu}}} = \frac{H}{2}.$$

[1]

Solving for x, we find:

$$x = \frac{H^2}{100}\frac{\rho U}{\mu} = \frac{(0.01\,m)^2}{100}\frac{1000\,\frac{kg}{m^3} * 0.2\,\frac{m}{s}}{0.001\,\frac{kg}{m*s}} = 0.2\,m$$

[3]

10. In Figure Q4 a visualization of a boundary layer over a flat plate is shown. Demonstrate using boundary layer theory, how an expression for the free stream velocity, U, of the fluid in which the plate is immersed can be estimated in terms of the boundary layer thickness, δ, at the end of the plate. [4 marks]

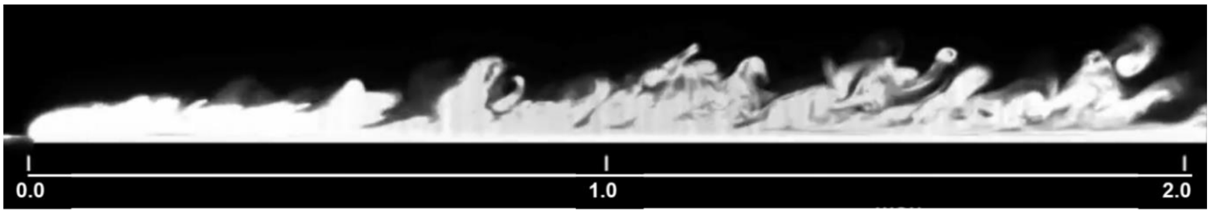


Figure Q.4: visualization of a boundary layer on a flat plate by Lee J.H. et al. The University of Melbourne.

Solution:

From the picture the boundary layer at the trailing edge of the plate is turbulent. [1]

To determine the free stream velocity it is necessary to

First step: Determine the thickness of the boundary layer at the trailing edge, from the picture. [1]

Second step: Use the following relationship from the solution of Prandtl equation for the Boundary Layer

$$\frac{\delta}{x} \approx \frac{0.16}{Re_x^{1/7}}$$

[1]

$$\frac{\delta}{x} \approx \frac{0.16}{\left({\rho U x}/{\mu}\right)^{1/7}}$$

hence U can be derived as:

$$U \approx \frac{0.16^7 \mu}{\rho x \left(\frac{\delta}{x}\right)^7} = \frac{2.68 \times 10^{-6} \mu x^6}{\rho \delta^7}$$

[1]

For using the laminar get first step, but can only get half second step (total max mark 2 of 4)

11. As a design engineer, you are required to study a wind turbine system. A model will be used in a wind tunnel to perform some experiments. The wind turbine will generate power when the blades are rotating at a rotational speed N. The surface of the prototype wind turbine, is to be considered smooth. A sketch of the wind turbine is illustrated in figure Q5. You are required to:

i) List all the parameters that might be relevant for this problem. [3 marks]

ii) State the number of dimensionaless groups that it is possible to derive. [1 mark]

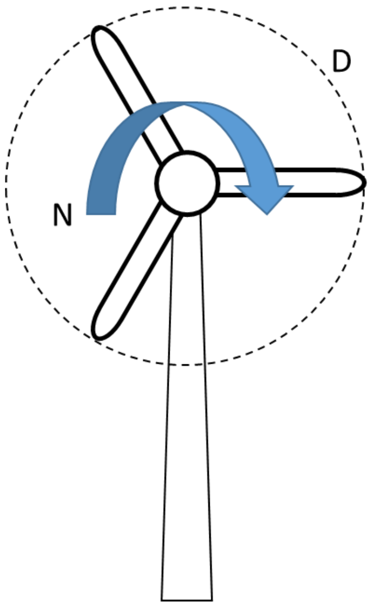


Figure Q5

Solution:

i) The variables are: i) P = power extracted; ii) D = diameter of wind turbine; iii) N = rotational speed of wind turbine; iv) U = velocity of the wind; v) ρ = density of the fluid; vi) μ = dynamic viscosity;

ii) The number of non-dimensional numbers are: 3 (6 variables and 3 dimensions). [1]

12. A flow situation is subjected to Buckingham Pi analysis, and the variables affecting the flow are density, ρ, gravity, g, length, L, velocity, U, dynamic viscosity, μ and surface area, A. With ρ, L, U as repeating variables, find the dimensionless group related to g and state its relevance.

Solution

ρ, L, U as repeating variables; μ, A and g will make groups. With 6 variables and 3 dimensions, we expect 3 dimensionless groups. But we focus on the one, g as follows:

dimensions of needed variables: g: LT^{-2}; ρ: ML^{-3}; L: L; U: LT^{-1}

dimensionless group is: $LT^{-2} \cdot (ML^{-3})^a \cdot L^b \cdot (LT^{-1})^c$

Therefore M: a=0 ; T: -2-c=0 $\rightarrow$ c = -2; L: 1-3a+b+c=0 $\rightarrow$ 1 + b - 2 = 0; b = 1

produces: gL/U^2

this is the Froude number inverted, which is the ratio of weight force to inertia force.